In productivity studies, the rates of technological change, which reflect changes in an industrial output for a given input bundle, may be measured by using a production function similar to that developed by Solow. Alternatively, rates of technological change which reflect changes in the cost of obtaining a given output may be measured by using a cost function such as that used in Ball and Chambers. This study applies a production function approach because it provides useful information for a direct explanation about the source of output growth.

The data used in this study are mainly compiled from the Bureau of the Census's *Annual Survey of Manufactures and Census of Manufacturing*. Two common production indicators, the value of shipments and the value added, are available in the data sources. This study uses both indicators as the output of a production function for measuring productivity indexes. The potential difference in productivity measurements from these production indicators is an issue addressed in this study. Following is a brief explanation of the methodology used to measure the multifactor and labor productivity indexes, and a method to modify the model for application to U.S. food manufacturing.

Derivation of Productivity Measures

In productivity studies, multifactor productivity is derived by taking account of various inputs into the productivity measurement. To measure the multifactor productivity index, the underlying production function is assumed to be Hicks' neutral technical change. The general form of the production function with n-factor inputs at time t can be written as:

$$Q_t = A_t f(X_{1t}, X_{2t}, \ldots, X_{nt}), \qquad (1)$$

where variables are Q_t (real output), X_{it} (input of the ith factor, i = 1, 2,..,n), and A_t (index of Hicks' neutral technical change or multifactor productivity). Although the assumption of neutral technical change may be rigid, this production function provides a framework for easy interpretation of the causes of productivity changes.

Differentiating equation (1) with respect to time t, the derived output growth equation becomes

$$(dQ_t/dt) / Q_t = (dA_t/dt) / A_t + \Sigma_i (\partial Q_t/\partial X_{it}) (X_{it} /Q_t) (dX_{it}/dt) / X_{it} \qquad (2)$$

Equation (2) shows the rate of change in output as the sum of the rate of change in multifactor productivity, $(dA_t/dt) / A_t$, and a weighted average of the rates of change in various inputs $(dX_{it}/dt) / X_{it}$. The weight is expressed by $(\partial Q_t/\partial X_{it}) (X_{it} /Q_t)$, which is the elasticity of output with respect to the ith input, showing the percentage change in output per 1-percent change in the ith input.

In addition, under the assumption that a competitive economy is operating at longrun equilibrium, the marginal products of all inputs are equal to their respective real market prices as $\partial Q_t / \partial X_{it} = W_{it} / P_t$, with new variables W_{it} (price of the ith input) and P_t (price of output). Substituting this expression for the elasticity of output in equation (2), and then using S_{it} (cost share of the ith input) to represent $W_{it} X_{it} / P_t Q_t$, the multifactor productivity index can be shown as:

$$(dA_t/dt) / A_t = (dQ_t/dt) / Q_t - \Sigma_i [S_{it} (dX_{it} /dt) / X_{it}] \qquad (3)$$

The competition in output markets indicates that the capital price reflects a competitive rate of return ensuring that all revenues are spent on inputs. In other words, the summation of all input cost shares equals 1 ($\Sigma_i S_{it} = 1$). Thus, the multifactor productivity index, showing the ability to produce more output from the same input, is calculated by subtracting an index series for the combined changes of various inputs from the index series for output changes. Different inputs are aggregated into one input measure by weighting (multiplying) the index series of each input by its share in the total cost of output.

Furthermore, the productivity index of the jth input can be shown as:

$$(dQ_t/dt) / Q_t - (dX_{jt}/dt) / X_{jt} = (dA_t/dt) / At + \Sigma_{i,\, i\neq j} S_{it} [(dX_{it}/dt) / X_{it} - (dX_{jt}/dt) / X_{jt}] \qquad (4)$$

In particular, if the jth input is regarded as labor, then this equation represents labor productivity. Accordingly, labor productivity, showing the rate of change in output per worker on the left-hand side of equation, is determined by two components: technological progress and the quantities of capital goods and other inputs available to each worker.

The Törnqvist Index Approximation

The rates of change in equations (3) and (4) are expressed in the Divisia index such as $(dQ_t/dt) / Q_t$ for the change of output and require using continuous data for the presentation. For empirical application, however, the Törnqvist index is commonly used as a discrete approximation of the Divisia index. More specifically, for example, the rate of change of output $(dQ_t/dt) / Q_t = (d\,ln Q_t/dt)$ can be approximated by $ln(Q_t/Q_{t-1})$. Similarly, the rate of change of the ith input $(dX_{it}/dt) / X_{it} = (d\,ln X_{it}/dt)$ can be approximated by $ln(X_{it}/X_{it-1})$. In addition, since the variables are expressed in consecutive change of observed data, an ideal weight S_{it} in the brackets of equations (3) and (4) should be the average shares of S_{it} and S_{it-1}; that is, $1/2(S_{it} + S_{it-1})$.

Therefore, by applying the Törnqvist index as a discrete approximation of the Divisia index, the multifactor productivity in equation (3) can be expressed as:

$$ln(A_t/A_{t-1}) = ln(Q_t/Q_{t-1}) - \Sigma_i \,[1/2(S_{it} + S_{it-1})\, ln(X_{it}/X_{it-1})] \qquad (5)$$

This expression shows that the rate of change of multifactor productivity $ln(A_t/A_{t-1})$ is the difference between the rate of change in output $ln(Q_t/Q_{t-1})$ and a weighted average of the rates of change of all factor inputs in the bracket. This methodology was used by the Bureau of Labor Statistics, and a discussion of the model for two factors (labor and capital) in a production function was documented in Mark and Waldorf.

Similarly, the Törnqvist index approximation of the productivity index of the jth input in equation (4) becomes:

$$ln(Q_t/Q_{t-1}) - ln(X_{jt}/X_{jt-1}) = ln(A_t/A_{t-1}) + \Sigma_{i,\ i\neq j}\, 1/2(S_{it} + S_{it-1})\,[ln(X_{it}/X_{it-1}) - ln(X_{jt}/X_{jt-1})] \qquad (6)$$

Again, if the jth input is regarded as labor, then this equation represents labor productivity. The above expression in natural logarithmic form shows that the rate of change of labor productivity is equal to the sum of the rate of change of multifactor productivity and the contribution of the changes in all other inputs per unit of labor to output.

While the above procedures for measuring productivity can be easily implemented, one might question that the underlying assumption of perfect competition may not be appropriate to the food manufacturing sector, which may be characterized by oligopoly. Ideally, we need to perform some tests on the potential oligopoly structure of the food manufacturing sector, but these tests are beyond the scope of this study. A noted paper by Azzam et al. incorporated information about markups, demand, and cost parameters into the measurement of productivity. This set of extraneous information, however, is obtained from different sources, and may introduce errors in the productivity measurement because the extraneous information is not obtained within the same framework as the measurement of productivity changes.

Empirical Modeling

In applying the methodology of measuring productivity for the U.S. food manufacturing industries, two commonly used output indicators (the value of shipments and the value added) are available in the *Census of Manufactures* and *Annual Survey of Manufactures*. By using these output indicators, this study applies two approaches to specify a production function for measuring the multifactor and labor productivity indexes.

One is the gross-output approach, such that the adjusted value of shipments is a function of capital, labor, energy, and material inputs as follows:

$$Q_t = A_t\, f(K_t, La_t, Lb_t, E_t, M_t), \qquad (7)$$

where Q_t is the gross output defined as the value of shipments adjusted by the net change in inventories measured at 1982 prices, with the producer price index of processed foods and feeds as a deflator. K_t represents capital services charges measured at 1982 prices, with the producer price index of capital equipment as a deflator. Capital services charges are approximated as the sum of depreciation charges for fixed assets and interest costs on the average value of fixed assets at the beginning and ending of that year. The labor inputs are divided into two components: production and nonproduction workers. La_t represents production worker-hours, and Lb_t is the number of nonproduction employees. E_t represents purchased fuels and electricity at 1982 prices, with the producer price index for intermediate energy goods as a deflator. M_t is the cost of materials at 1982 prices, with the producer price index of crude foodstuffs and feedstuffs as a deflator. A_t is the index of multifactor productivity for the value of shipments. This gross-output production function represents a production structure that includes the contribution of all factor inputs that are available in the data sources.

The net-output approach uses net output or the net value added as an output in a production function. Net output is calculated by subtracting the cost of materials, supplies, containers, fuel, electricity, and purchased services from the value of shipments and then deflating by the producer price index of processed foods and feeds. The net output represents the value that is added, by the application of capital and labor, to intermediate inputs in converting those inputs to finished products. Therefore, capital and labor are the relevant inputs in generating the net output of an industry, and a production function for the net output is specified as follows:

$$Q_t^* = A_t^* f(K_t, La_t, Lb_t), \qquad (8)$$

where Q_t^* is the quantity of net output or net value added, and K_t, La_t, and Lb_t are defined the same as in equation 7. A_t^* is the index of multifactor productivity for the net value added.

The existence of this net-output production function, as discussed in Gullickson, requires that the production of gross output (as shown in equation 7) be characterized by value-added separability, in which intermediate inputs cannot be the source of productivity growth. In other words, intermediate inputs are excluded from consideration in the net-output model on the assumption that they are insignificant to the analysis of productivity growth. With this restrictive assumption, the purpose of measuring net-output productivity from equation 8 is to calculate an industry's contribution to the Nation's GDP in a simple and straightforward way. For interpreting industry productivity, however, the gross-output model specification is generally preferred.