# Appendix 1: Techniques for Measuring Risk

***Understanding how risk is measured is a starting point for helping farmers make choices about the most appropriate strategies for their individual situations. This appendix provides information on the different approaches that can be used to quantify risk, and illustrates how probability distributions can be used to characterize the outcomes associated with risky choices. In order to make the best decisions for their individual operations, farmers and other decision-makers often use historical and current information about prices, yields, weather conditions, and other variables to estimate future risk.***

Risk must be quantified in order to evaluate whether various risk management tools and strategies are effective in achieving producers' risk reduction goals. This process involves measuring uncertainty and quantifying the relationship between uncertainty and an individual's well being. This section discusses how risks can be quantified and provides representative estimates for selected locations—focusing on price variability, yield variability, and the correlation between prices and yields (the extent to which prices and yields move together).

## Measuring Uncertainty

The measurement of uncertainty involves estimating the probabilities of future outcomes. Estimates may be made, for example, of the probability of yield less than 100 bushels per acre, the probability of price falling below $2.25 per bushel, or the probability of revenue less than $200 per acre. More generally, one would like to estimate the joint probability distribution of yield, price, and revenue so that one might, for example, specify the probability of revenue falling below any specified level. To estimate such probabilities, we generally start by observing historical outcomes and separating random variability from systematic variability.

To illustrate, appendix figure 1 (and appendix table 1) show corn yields for Iroquois County, Illinois (in the east central part of the State) for the years 1956-95. The jagged line links the actual yields, averaged across the county, for each year, while the straight line represents the systematic upward trend in yields. This upward trend may be considered to be a "known" source of variation that will repeat itself in the future. It has been caused by several factors, including the development of higher yielding varieties and the introduction of improved chemicals and fertilizers. In contrast, the yield deviations from trend—mainly caused by weather—constitute the random variability.[31]

Quantifying yield randomness generally involves summarizing what

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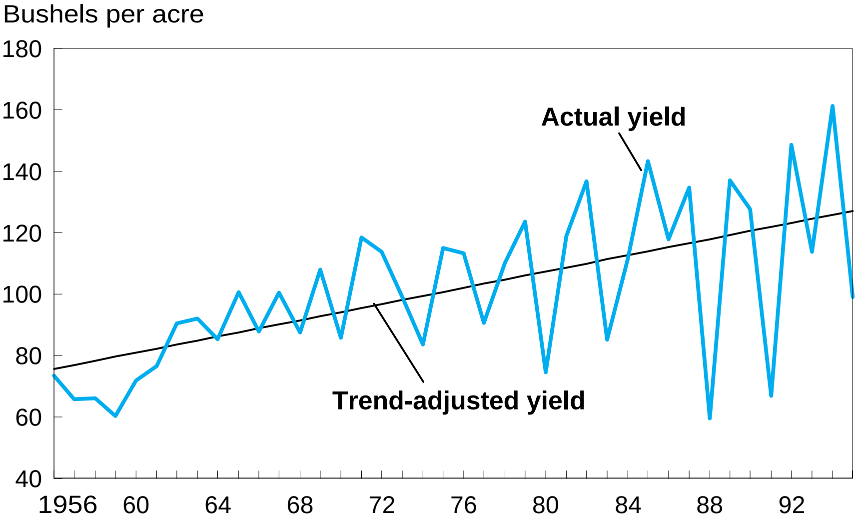
[31] Whether the yields shown are adequately represented by a linear trend can be questioned. Linear yield trends often are used for forecasting, but there is no strong reason why yield trends should be linear, or follow any other specific mathematical form. Trend projections inevitably involve a degree of subjectivity, not only in choosing the mathematical function to use, but also in selecting the years to be included in calculating the trend.

Appendix figure 1

### Actual and trend-adjusted corn yields, Iroquois County, Illinois, 1956-95



Note: Actual yield is county average.

Source: Constructed by ERS from USDA, NASS electronic county yield files, 1997.

is known about deviations from expected yields, as measured by trend. Randomness can be described by converting such deviations into a frequency distribution, or histogram, as depicted in appendix figure 2.[32] Each bar on the figure shows the number of times that yield deviations from trend in Iroquois County fell within a particular 10-bushel-per-acre range. For example, the bar labeled "-5 to +5" illustrates that yields fell between -5 bushels and +5 bushels from trend in 7 of the 40 years between 1956 and 1995, and the bar labeled "5 to 15" illustrates that yields fell between 5 bushels and 15 bushels above trend in 9 of the years. Frequencies are greatest near the middle and the least at the lower and upper ends, which is typical of yields, prices, and revenues. This is because extreme weather events—such as the 1988 drought—are less likely than weather events having a more modest effect.

The degree of randomness is reflected in the width of the distribution and in the number of observations that are distant from the mean. Note that appendix figure 2 is not symmetrical (like the traditional bell curve), but that the lower tail is longer than the upper tail. This so-called negative skewness is typical of yield distributions. This shape occurs because devastating weather can cause very significant yield declines (as low as zero), while very good weather is likely to only moderately boost yields above trend due to the physiological limitations of the plant.

For many purposes, a single number is a more convenient measure of randomness (or dispersion) than is an entire distribution. The most widely used measures of randomness are the variance and its square root, the standard deviation. Variance is the average squared deviation from the mean, or trend. By using the variance of deviations from trend, a large part of the systematic variation is removed.

One problem with the variance and standard deviation is that

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[32] To construct appendix figure 2, randomness was assumed to have remained unchanged, although appendix figure 1 suggests that it may be increasing over time.

**Appendix table 1—Calculation of yield variability for Iroquois County, Illinois**

| Year | Actual yield | Trend-adjusted yield | Deviation (actual minus trend) |
|---|---|---|---|
| | | *Bushels per acre* | |
| 1956 | 73.4 | 75.7 | -2.2 |
| 1957 | 65.8 | 77.0 | -11.1 |
| 1958 | 66.1 | 78.3 | -12.2 |
| 1959 | 60.2 | 79.6 | -19.4 |
| 1960 | 71.8 | 80.9 | -9.1 |
| 1961 | 76.6 | 82.3 | -5.7 |
| 1962 | 90.4 | 83.6 | 6.9 |
| 1963 | 92.0 | 84.9 | 7.1 |
| 1964 | 85.3 | 86.2 | -0.9 |
| 1965 | 100.6 | 87.5 | 13.0 |
| 1966 | 87.8 | 88.9 | -1.0 |
| 1967 | 100.4 | 90.2 | 10.2 |
| 1968 | 87.6 | 91.5 | -3.9 |
| 1969 | 108.0 | 92.8 | 15.2 |
| 1970 | 85.8 | 94.1 | -8.3 |
| 1971 | 118.6 | 95.5 | 23.1 |
| 1972 | 113.8 | 96.8 | 17.0 |
| 1973 | 99.1 | 98.1 | 1.0 |
| 1974 | 83.5 | 99.4 | -15.9 |
| 1975 | 115.0 | 100.7 | 14.2 |
| 1976 | 113.3 | 102.1 | 11.2 |
| 1977 | 90.7 | 103.4 | -12.7 |
| 1978 | 110.1 | 104.7 | 5.4 |
| 1979 | 123.6 | 106.0 | 17.5 |
| 1980 | 74.6 | 107.3 | -32.8 |
| 1981 | 118.9 | 108.7 | 10.2 |
| 1982 | 136.7 | 110.0 | 26.7 |
| 1983 | 85.3 | 111.3 | -26.0 |
| 1984 | 111.4 | 112.6 | -1.3 |
| 1985 | 143.2 | 113.9 | 29.3 |
| 1986 | 117.9 | 115.3 | 2.6 |
| 1987 | 134.7 | 116.6 | 18.1 |
| 1988 | 59.6 | 117.9 | -58.3 |
| 1989 | 137.1 | 119.2 | 17.8 |
| 1990 | 127.6 | 120.5 | 7.0 |
| 1991 | 66.9 | 121.9 | -54.9 |
| 1992 | 148.6 | 123.2 | 25.5 |
| 1993 | 113.7 | 124.5 | -10.8 |
| 1994 | 161.2 | 125.8 | 35.4 |
| 1995 | 99.0 | 127.1 | -28.1 |

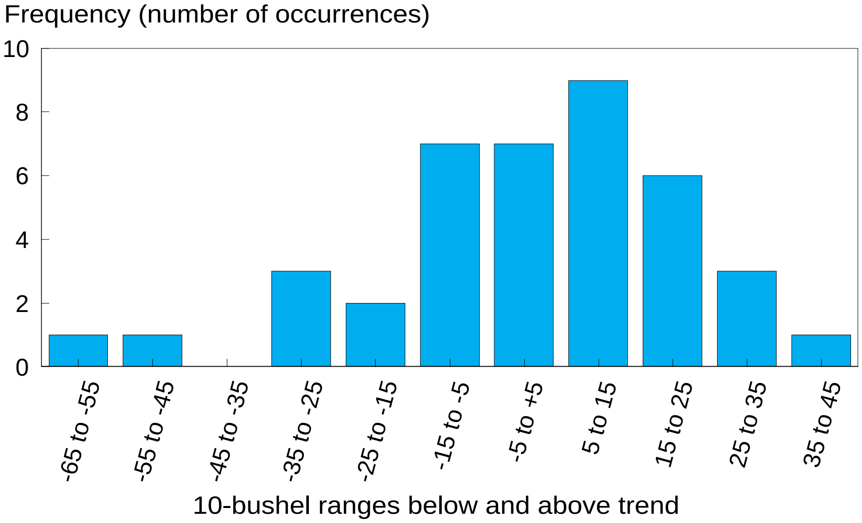
Note: The equation estimated from these data for detrending yields is: $E(Y_t) = 1.72 + 1.32(T)$, where T is the year, minus 1900.
Source: Calculations made by ERS from USDA, NASS, electronic county yield files, 1997.

they are difficult to interpret without knowing the level or magnitude of the underlying variable. A variance of 10 bushels, for example, has quite different implications for the tightness of the distribution when the mean yield (adjusted for trend) is 50 bushels per acre than when it is 160 bushels. As a result, proportional variability—or variability relative to the mean—is often measured to facilitate comparisons. The most commonly used measure of relative variability is the coefficient of variation, which equals the standard deviation divided by the mean.

The variance (or alternatively, the standard deviation or coefficient of variation) is a good measure of variability for approximately symmetric, bell-shaped distributions. It fully describes the variability in a normal distribution, which is a particular bell-shaped mathematical distribution that closely approximates many observed distributions. Most yield distribu-

Appendix figure 2

**Frequency distribution of corn yield deviations from trend, Iroquois County, Illinois, 1956-95**



Source: Constructed by ERS from USDA, NASS electronic county yield files, 1997.

tions, however, appear to be non-normal with long lower tails as shown in appendix figure 2. Moreover, some tools used to manage farmers' risks, particularly crop insurance and commodity options, impose non-normality by setting bounds or limits on the lower tails of the yield or price distributions realized by the farmer. Producers generally prefer yield and price distributions that are bounded from below because it limits their losses. However, the standard deviation may not provide a satisfactory measure of risk under such distributions, which clearly are non-normal.

Other measures of variability or dispersion may be useful for distributions that are clearly non-normal. One such measure is the probability of outcomes below some critical level. The probability of yield less than 70 percent of its expectation, for example, might be a useful measure of risk for some farmers. If the trend yield is 127 bushels per acre, the 70 percent point would equal 0.70 * 127, or 89 bushels per acre. This is 38 bushels below trend. In appendix figure 2, the probability of such a yield (or lower) is two occurrences in 40 years, or a probability of 2/40 = .05. Individual farmers might choose higher or lower cutoff points, depending on their differing financial circumstances and degrees of risk aversion.

## Estimating Probabilities of Future Events

Farmers, like other decisionmakers, are fundamentally concerned with randomness in future events, not the distribution of past outcomes as illustrated in the previous section. They are concerned about the probabilities of outcomes to be observed in the future and the effects of these outcomes on their economic welfare. The probability associated with any given outcome indicates the strength of one's belief that such an outcome will occur, ranging from zero (which represents no possibility) to 1 (representing absolute certainty).

Two sources of information about such probabilities are available: logic and experience. In pure games of chance, logic rules. For example, in flipping a coin, two equally likely outcomes are possible—heads or tails—and thus a probability of 0.5 can be assigned to each. In business decisions, however, historical observations often must be relied upon to estimate probabilities. Each of the frequencies illustrated in appendix figure 2, for instance, could be divided by the total number of years, 40, to obtain estimates of the probabilities of yields within each of the intervals. The resulting distribution is often referred to as an "empirical" probability distribution because it is estimated through the use of a specific set of historical observations. Suppose that the projected mean yield is 130 bushels per acre. Referring to appendix figure 2, the estimated probability of the yield falling between 115 and 145 bushels (that is, between -15 and +15 bushels from the trend expectation) is 23/40 = 0.575.

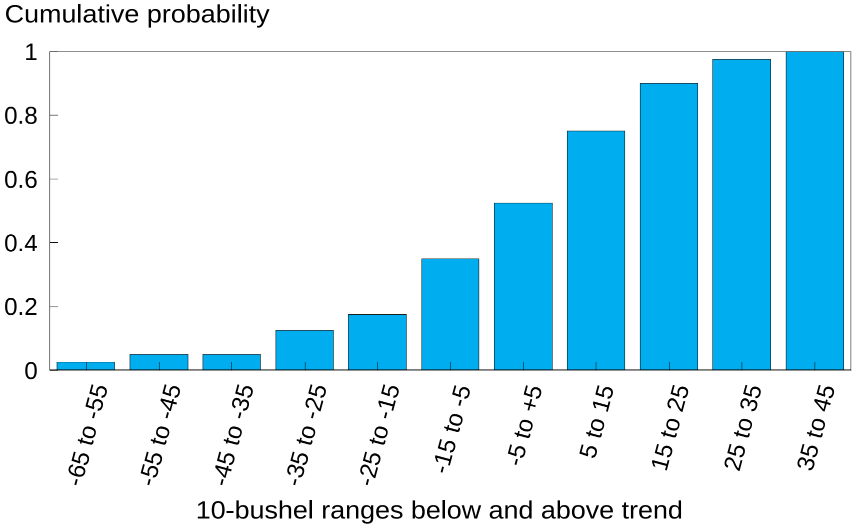
An alternative way to describe dispersion graphically is to plot probabilities of outcomes falling at or below specific values. This is called a cumulative distribution. Appendix figure 3 is a cumulative distribution of the Iroquois County yield deviations. Cumulative distributions are particularly useful for representing continuous variables because probabilities can be read directly from the vertical axis instead of by summing areas under a curve. Cumulative distributions are useful in safety-first analysis and stochastic dominance analysis, which are discussed in the next section of this report.

Relative frequencies derived from historical observations are not necessarily the best estimates of future probabilities. Sometimes, the decisionmaker has additional information—such as regarding recent rainfall or temperature conditions—which needs to be taken into account. Moreover, most historical series include events that have small probability of recurring, or fail to catch events, that though uncommon, have a non-zero likelihood. To reduce the impacts of such sampling errors, forecasters often impose smoothness and a degree of symmetry by fitting mathematical distributions to historical observations.

The normal distribution, which is symmetrical and bell-shaped, is frequently used as an approxima-

Appendix figure 3

### Cumulative probability of corn yield deviations from trend, Iroquois County, Illinois, 1956-95

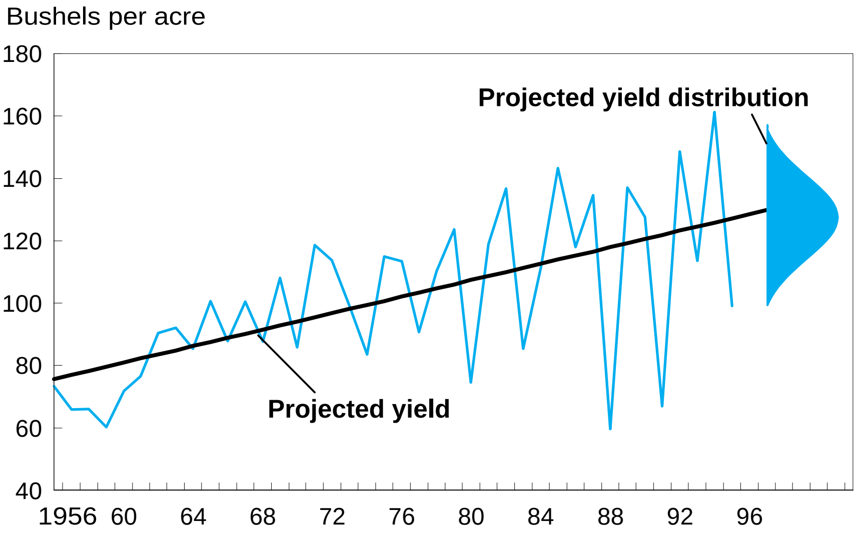


Source: Constructed by ERS from USDA, NASS electronic county yield files, 1997.

tion. Although yield distributions are typically negatively skewed, as discussed earlier, the normal distribution is computationally convenient because it is fully described by its mean and variance. In addition, yield deviations from normality may not be great. The mean and variance in appendix table 1, for example, can be used as parameters of a normal distribution of yield deviations from trend. Appendix figure 4 illustrates realized corn yields for Iroquois County over the 1956-95 period and a projected probability density function for the 1997 yield. The projected distribution reflects the belief that the true probability function is continuous and recognizes that observed historical observations between 1956 and 1995 are only a sample of the possible outcomes.

Appendix figure 4

**Projected 1997 corn yield distribution for Iroquois County, Illinois, based on 1956-95 observations**



Source: Constructed by ERS from USDA, NASS electronic county yield files, 1997.